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O.R. Applications

Lagrangian relaxation based algorithm for trigeneration planning with storages

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Abstract

Trigeneration is a booming power production technology where three energy commodities are simultaneously produced in a single integrated process. Electric power, heat (e.g. hot water) and cooling (e.g. chilled water) are three typical energy commodities in the trigeneration system. The production of three energy commodities follows a joint characteristic. This paper presents a Lagrangian relaxation (LR) based algorithm for trigeneration planning with storages based on deflected subgradient optimization method. The trigeneration planning problem is modeled as a linear programming (LP) problem. The linear cost function poses the convergence challenge to the LR algorithm and the joint characteristic of trigeneration plants makes the operating region of trigeneration system more complicated than that of power-only generation system and that of combined heat and power (CHP) system. We develop an effective method for the long-term planning problem based on the proper strategy to form Lagrangian subproblems and solve the Lagrangian dual (LD) problem based on deflected subgradient optimization method. We also develop a heuristic for restoring feasibility from the LD solution. Numerical results based on realistic production models show that the algorithm is efficient and near-optimal solutions are obtained.

Keywords: Linear programming; Lagrangian relaxation; Trigeneration; Energy storage; Deflected subgradient optimization

1. Introduction

With the competitive and economic pressures to cut expenses, increase air quality, and reduce emissions of air pollutants and greenhouse gasses, the trigeneration power and energy system is becoming a preferred method to produce clean energy and power for buildings (school, offices, hotel, shopping centers, and hospital) and manufacturing plants. Trigeneration is the conversion of the primary energy source (fuel) into three

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useful energy commodities in a single integrated process. Three energy commodities can take different forms based on the specific applications and the production of the energy commodities follows a joint characteristic. Trigeneration is typically defined as the simultaneous production of electric power, heat and cooling. Trigeneration takes combined heat and power (CHP, simultaneous production of heat and power) technology one step by utilizing the waste heat in process to produce cooling. Many industries and commercial buildings need trigeneration. For example, trigeneration of power, steam or thermal oil, and chilled water for the plastic and rubber industry, trigeneration of quality UPS (uninterruptible power supply), steam and chilled water for the semiconductor industry and trigeneration of power, steam/hot water and chilled water for a district heating and cooling system (Active, 2000). Trigeneration can achieve higher total energy efficiency than what is possible by producing the three commodities separately or by CHP technology. Therefore, it can save both fuel and emissions. Goodell (2002) illustrated using a simple trigeneration plant prototype that trigeneration can save about 24.5% of primary energy than CHP.

Besides trigeneration plants, a trigeneration system may contain CHP plants and plants for producing different energy commodities separately such as condensing plants, hydropower, heat plants, cooling devices as well as various purchase and sale contracts for externally produced energy. Most relevant for this paper, we consider possible energy storages (hot water and chilled water tanks) for two other commodities except power in the system. The incorporation of energy storage can increase the flexibility of the system operation. Under the deregulated power market, power is produced to respond to power market price and two other energy commodities are produced to satisfy demands. The storages can be used to maximize the power production in CHP plants and trigeneration plants when power price is high and they can also be used to minimize the use of plants with higher operational costs. That is, the storage devices can store the produced energy during cheap periods and release the stored energy during expensive periods.

The joint characteristic of the trigeneration makes the operating region of trigeneration plants three-dimensional, which is more complicated than that of CHP (two-dimensional) and power-only generation. The power-only generation system and CHP system can be viewed as a special case of trigeneration systems (Rong and Lahdelma, 2005c; Rong, 2006). Particularly, under the deregulated market, CHP system can be reduced to one-dimensional system (heat) with power generation responding to power price (Rong and Lahdelma, 2007a). Similarly, the trigeneration system can be reduced to two-dimensional systems. The determination of the feasible region is trivial for one-dimensional line but elaborate for two-dimensional area.

The incorporation of energy storage introduces dynamic constraints in the system. Based on different application backgrounds and systems, various approaches have been proposed to solve these types of problems. Here our emphasis is placed on the approaches for dealing with *storage constraints*. Based on the extensive survey of literature, many publications address the energy storage (pumped-storage) for power-only generation systems such as hydro or hydro-thermal systems and some address energy storage (chilled or hot water tank) for CHP systems. The solution approaches follow two lines. The first line decouples the time-dependent storage constraints and the solution of the overall multi-period large-size problem is reduced to the solution of multiple small-size well-structured single-period subproblems. The solution approaches include Lagrangian relaxation (LR) methods (Dotzauer and Ravn, 2000; Guan et al., 1994; Ngundam et al., 2000) and dynamic programming (DP) methods (Ferrero et al., 1998; Gorenstin et al., 1992; Korpaas et al., 2003; Pereira, 1989; Yang and Chen, 1989). The second line treats the multi-period storage problem (subproblem) as an entity and solves it by Simplex method (Bos et al., 1996), interior point method (IPM) (Medina et al., 1997) and network flow method (Ferrerira et al., 1989). The application of the evolutionary programming (Lai et al., 1998) should also be classified in this category.

In this paper, we consider the medium- and long-term trigeneration planning with storages under the deregulated power market without unit commitment (UC) involvement. This problem specification is useful in risk analysis in conjunction to long-term strategic decision-making. In this context, it is acceptable that the UC sub-problems are solved approximately using some heuristics (Voorspools and D'haeseleer, 2003) and thus we can place the emphasis on the economic dispatch problem (Rong and Lahdelma, 2007c). The introduction of the competitive market implies that the actors such as producers, traders, distributors and end customers in the market are exposed to substantial risks caused by volatile market situations. A simple and effective method for risk analysis based on the analysis of randomly generated scenarios of power price and energy demand profiles (Breipohl et al., 1994; Makkonen and Lahdelma, 1998, 2001; Rong and Lahdelma, 2005b, 2007b).

In each scenario, a deterministic long term planning model is solved. With randomly generated simulation scenarios, the robustness and speed of algorithms for solving deterministic problems are imperative. Under such application context, the decoupling technique is favorable because the size of the problem can become very large with long planning horizon and the computational requirements are expensive for directly solving the large-size problem.

Here we deal with trigeneration planning problems (Rong and Lahdelma, 2005a,c; Rong, 2006) with storages based on the LR decomposition framework. The LR procedure decomposes the original problem into multiple underlying Lagrangian subproblems (LS). Based on different application backgrounds, the LS of the integrated energy production (cogeneration of more than one energy commodities) planning problems can be represented as linear programming (LP)/mixed integer linear programming (MILP) models (Dotzauer, 2003; Gardner and Rogers, 1997; Grohnheit, 1993; Lahdelma and Hakonen, 2003; Lahdelma and Rong, 2005; Makkonen, 2005; Rong, 2006; Rong and Lahdelma, 2005c) or non-linear programming (NLP) (e.g. quadratic programming) models (Dotzauer, 2001; Dotzauer and Ravn, 2000; Song et al., 1999). We adopt the LP-based (including MILP) modeling techniques (Rong and Lahdelma, 2005a,c; Rong, 2006). The benefit of LP-based models is that we have reliable and quite efficient algorithms such Simplex algorithms (Dantzig, 1963) and IPM (Karmarkark, 1984) for solving the underlying generic LP problems. In conjunction with the specialized modeling techniques for some application contexts, we have also developed extremely efficient algorithms for solving the structured LP/MILP problems (Lahdelma and Hakonen, 2003; Makkonen and Lahdelma, 2006; Rong and Lahdelma, 2005a, 2006, 2007a; Rong et al., 2006). To solve the large-scale LP-based problems in the complicated settings (dynamic constraints) efficiently, it is desirable to utilize efficient LP solvers intelligently.

However, LP problems pose challenge for the LR technique because linear cost function can result in a rather slow and unsteady convergence (Sherali and Ulular, 1989) when the pure subgradient method is used to solve the Lagrangian dual (LD) problem. There are several alternatives to overcome this difficulty. Firstly, the augmented LR technique (Beltran and Heredia, 2002; Wang et al., 1995) is used by introducing quadratic penalty terms to the objective function. Secondly, the non-linear approximation (quadratic function) (Guan et al., 1995) is used to approximate the linear function. Thirdly, the bundle method (BM) (Redondo and Conejo, 1999; Zhang et al., 1999) and analytical center cutting plane method (ACCPM) (Gondzio et al., 1997) are used to generate the search directions better than those by the subgradient method. All of these methods can enhance the convergence of the algorithm. However, the first two methods make the efficient LP solvers invalid because of the introduction of the quadratic terms. BM and ACCPM require much more computational effort to obtain the proper search directions.

In this paper we enhance the convergence of the algorithm for solving the LD problem by using a computationally-efficient deflected subgradient method (Sherali and Ulular, 1989) in conjunction with the proper transformation of storage constraints for forming the LS based on the problem characteristics. We do not introduce any penalty terms in forming the LS. The resultant LS can be solved using any generic LP solvers as well as the efficient specialized tri-commodity simplex (TCS) algorithm (Rong and Lahdelma, 2005a). Finally, we should observe that almost all of the LR algorithms except a few like the volume algorithm (Barahona and Anbil, 2000) may generate an infeasible LD solution because of the relaxed constraints. It is not a trivial issue to restore the feasibility for our problem because of the joint characteristics of the trigeneration plant. To obtain a good feasible solution, we develop a heuristic considering the loss factor, charge/discharge efficiency of the storage and the feasible region of the system that must be constructed explicitly.

2. Problem formulation

In most cases, trigeneration can be viewed as a technology that takes CHP production one step by utilizing the waste heat in the process to produce cooling (e.g. chilled water) for air conditioning or industrial process. Utilization of trigeneration technology can result in significant energy savings when all of the three commodities are used. Under the deregulated power market, we assume that energy producers are price takers. With the term price-taker, we refer to any firm that has slight weight to change the market price by means of its offers. That is, the price clearing process is represented as exogenous to the company's optimization program. The planning of a trigeneration system is based on hourly load forecast for heat and cooling as well as power price. The hourly production level in the plants should be adjusted based on power price, demands for two other commodities and production cost.

The key to successful trigeneration is to design the plant according to the demands of heat and cooling. The primary concern of the trigeneration is to produce heat and cooling to satisfy variable demands. A trigeneration system may include trigeneration plants, CHP plants and plants for producing different energy commodities separately to make the system more flexible and reliable under variable demands. Incorporation of energy storages such as hot/chilled water tanks can further increase the flexibility of system operation. Since the production of different energy commodities in the trigeneration and CHP plants follows a joint characteristic, the storage devices can be used to maximize power production in trigeneration and CHP plants during peak power price periods and they can also used to minimize the use of the plants with higher operational costs. That is, the storage devices can enable the storage of produced heat/cooling during the cheap periods and release of the stored heat/cooling during the expensive periods. The objective function of the trigeneration system is to minimize the overall *net acquisition costs* for power and other energy commodities. The net acquisition costs consist of actual production costs (fuel costs), costs for purchasing components minus revenue from selling the produced energy.

2.1. Modeling a generic trigeneration plant

For a generic trigeneration plant model, we use the generic cogeneration plant (simultaneous production two or more energy commodities) model presented by Rong (2006) and Rong and Lahdelma (2005c). The modeling technique is the extension of the CHP plant model (Lahdelma and Hakonen, 2003; Makkonen and Lahdelma, 2006) to accommodate any number of commodities. The joint characteristic of the trigeneration plant can either be convex or non-convex. The convex characteristic means that characteristic operating region of the plant is convex in terms of three energy commodities (p, q, r) (e.g. power, heat and cooling) and the production cost c is a convex function of the generated energy commodities. Then the convex characteristic operating region can be represented as a *convex combination* (see e.g. Bazaraa and Shetty, 1993) of extreme points (c_j, p_j, q_j, r_j) (cost, power, heat and cooling) that define the region. An example of the extreme points for the traditional generic backpressure plant is given in Table 1.

For the advanced production technologies such as gas turbine and combined gas and steam cycles, the characteristic operating region may be non-convex. A non-convex characteristic can be divided into multiple convex sub-regions, which are encoded as alternative model components (Makkonen and Lahdelma, 2006; Rong, 2006; Rong and Lahdelma, 2005c, 2006). The same modeling technique applies also to other energy acquisition components, such as separate heat, cooling and power plants, purchase contracts, and

Table 1
Extreme points of a generic backpressure plant

c	p	q	r
732.11	80.62	1.56	200
632.58	72.73	0	181.82
732.11	68.51	171.28	0
732.11	62.43	200	0
732.11	52.38	135	200
732.11	45.5	200	142.22
390	45.16	112.89	0
390	39.95	137.5	0
390	33.04	92.81	137.5
390	28.31	137.5	97.78
108.42	0	34.62	24.62
97.89	0	20.77	30.77
86.13	0	26.47	0
71.3	0	0	21.05

c – cost, p – power, q – heat, r – cooling.

demand-side-management components. Consequently, the convex and non-convex characteristic can be modeled as linear programming (LP) and mixed integer linear programming (MILP) model respectively. When the decomposition techniques such as Lagrangian relaxation (LR) method are used to solve the problem, the overall problem decomposes into hourly models for solution. Therefore, the overall solution approach would be same for both convex and non-convex model and only the solvers for solving the hourly model are different. That is, LP/MILP solvers are used for solving the hourly convex/non-convex models respectively. In the following system model, we assume that the plant characteristic is convex.

2.2. Modeling trigeneration system

The following notations are introduced to formulate the problem.

p, q, r	Super/subscripts or prefixes for three energy commodities in the system
t	Refer to either a period or a point in time. The period t is between point $t-1$ and t. In our problem, one period is one hour
T	Number of periods over the planning horizon

Index sets

B	Set of q and r-commodity, i.e. $B=\{q,r\}$. The major purpose of introducing this set is to simplify the representation because of the symmetry of q and r-commodity
J_u	Set of extreme points of the operating region of plant u
J_t	Set of extreme points of the operating regions of all plants committed in period t. $(J_t=\bigcup_{u\in U_t} J_u)$
U_t	Set of all plants committed in period t

Parameters

$(c_{j,t}, p_{j,t}, q_{j,t}, r_{j,t})$	Extreme point $j \in J_u$ of operating region of plant u (cost, power, q-commodity, r-commodity) in period t
$c_{p,t}$	Power price on market in period t
$h_{b,s}, h_{b,e}$	Initial and terminal b-storage level, $b \in B$
$\underline{h}_b, \bar{h}_b$	Minimum and maximum b-storage level, $b \in B$
Q_t, R_t	q- and r-commodity demand in period t
$\underline{x}_{b\pm}, \bar{x}_{b\pm}$	Minimum and maximum charge/discharge rate of b-storage (MW/hour), $b \in B$
ρ_b	Fraction loss for b-storage, $b \in B$
$\eta_{b\pm}$	Charge/discharge efficiency of b-storage, $b \in B$

Decision variables

$x_{j,t}$	Variables encoding the operating level of each plant in terms of extreme points in period t, $j \in J_t$
$x_{p,t}$	Net power level in period t
$x_{b\pm,t}$	Charge/discharge rate of b-storge in period t, $b \in B$

State variables

$h_{b,t-1}$	b-storage level at the beginning of period t, $b \in B$.

Then the trigeneration planning with storages can be formulated as follows.

$$\text{Min} \quad \sum_{t=1}^{T}\left(\sum_{j\in J_t} c_{j,t}x_{j,t} - c_{p,t}x_{p,t}\right) \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in J_u} x_{j,t} = 1, \quad u \in U_t, \quad t = 1,\ldots,T, \tag{2}$$

$$\sum_{j\in J_t} p_{j,t}x_{j,t} - x_{p,t} = 0, \quad t = 1,\ldots,T, \tag{3}$$

$$\sum_{j\in J_t} q_{j,t}x_{j,t} - x_{q+,t} + x_{q-,t} = Q_t, \quad t = 1,\ldots,T, \tag{4}$$

$$\sum_{j\in J_t} r_{j,t}x_{j,t} - x_{r+,t} + x_{r-,t} = R_t, \quad t = 1,\ldots,T, \tag{5}$$

$$h_{b,t} = (1-\rho_b)h_{b,t-1} + \eta_{b+}x_{b+,t} - \frac{x_{b-,t}}{\eta_{b-}}, \quad b\in B, \quad t = 1,\ldots,T, \tag{6}$$

$$h_{b,0} = h_{b,s}, \quad b\in B, \tag{7}$$

$$h_{b,T} = h_{b,e}, \quad b\in B, \tag{8}$$

$$\underline{h}_b \leqslant h_{b,t} \leqslant \bar{h}_b, \quad b\in B, \quad t = 1,\ldots,T-1, \tag{9}$$

$$\underline{x}_{b\pm} \leqslant x_{b\pm,t} \leqslant \bar{x}_{b\pm}, \quad b\in B, \quad t = 1,\ldots,T, \tag{10}$$

$$x_{j,t} \geqslant 0, \quad j\in J_t, \quad t = 1,\ldots,T. \tag{11}$$

In this formulation, the convex combination for each plant is encoded by a set of $x_{j,t}$ variables, indicating the operating level of each plant in terms of extreme points of the operating region, whose sum is one (2) and that are non-negative (11). The power balance (3) determines the net amount of power $x_{p,t}$ that can be traded on the market at price $c_{p,t}$. Constraints (4) and (5) state the energy balances for q- and r-commodity in the system considering the charge/discharge of the energy storages. Constraints (6) are storage dynamics of q- and r-commodities. Constraints (7)–(10) are terminal conditions or boundary constraints associated with storages. If only the storage of one commodity exists, then only the dynamics and constraints addressing the corresponding commodity are active.

In the above model, the objective function (1) with constraints (2)–(5) and constraints (10), (11) forms the trigeneration planning model without dynamic constraints under the deregulated power market (Rong and Lahdelma, 2005a).

3. Constructing feasible operating region of trigeneration system

Under the deregulated power market, the power production responds to price and the production of q- and r-commodity satisfies demands. For the LR-based technique to solve the planning problem by relaxing q- and r-storage constraints, we need to know the feasible region defined by q- and r-commodity for the system to obtain the feasible solution from the LD solution. For the power-only system or the CHP system (under the deregulated market) the feasible region is trivially one-dimension line. For the trigeneration system, the feasible region of the trigeneration plant in terms of q- and r-commodity generation can be a two-dimensional area or one-dimensional line based on the specific plant characteristic as shown in Fig. 1. The points in Fig. 1 represent the projection of the extreme points of a trigeneration plant in (q,r)-plane.

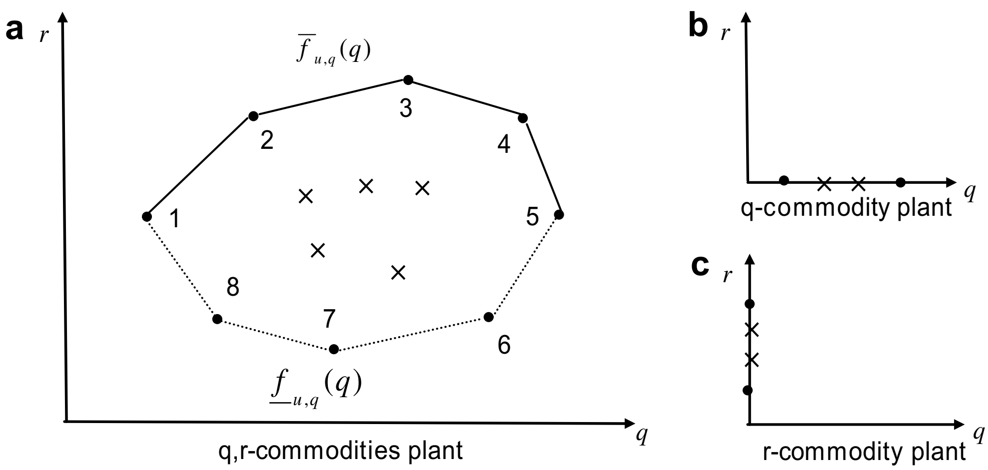

Fig. 1. The feasible region of a trigeneration plant u in (q,r)-plane.

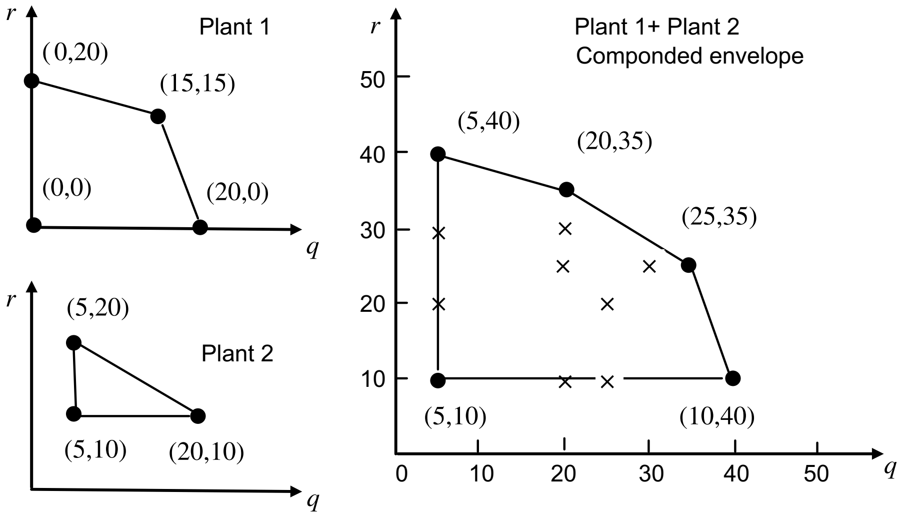

Fig. 2. Compounded envelope of two plants. The points marked in either crosses or dots in the compounded envelope are compounded points during the compounded process.

For a plant u which produces q- and r-commodity simultaneously (Fig. 1a), we define $r = \underline{f}_{u,q}(q)$ and $r = \bar{f}_{u,q}(q)$ as the lower and upper envelopes of (q, r)-region in terms of q. The points with cross marks are those not on the envelope and the points with dot marks are those on the envelopes. $\underline{f}_{u,q}(q)$, which consists of points 1, 2, 3, 4, and 5, is a piecewise linear convex function in terms of q. $\bar{f}_{u,q}(q)$, which consists of points 1, 8, 7, 6, and 5, is a piecewise linear concave function in terms of q. Similarly, we can define $q = \underline{f}_{u,r}(r)$ and $q = \bar{f}_{u,r}(r)$ as the lower and upper envelopes of (q, r)-region in terms of r. $\underline{f}_{u,r}(r)$ consists of points 7, 8, 1, 2 and 3 and $\bar{f}_{u,r}(r)$ consists of points 7, 6, 5, 4 and 3. For a given q, $[\underline{f}_{u,q}(q), \bar{f}_{u,q}(q)]$ is the feasible interval for r-commodity; for a given r, $[\underline{f}_{u,r}(r), \bar{f}_{u,r}(r)]$ is the feasible interval for q-commodity.

For a plant that does not produce q- and r-commodity simultaneously (Fig. 1b and c), the feasible region is trivially one-dimensional line. This situation is similar to that for the CHP system. The feasible interval is determined by the extreme points with minimum and maximum coordinate values for the corresponding commodity. There are only two points on the envelope. We view this special situation as either the upper or lower envelopes coincide or only the lower envelope exists. The power-only component does affect the operating region in (q, r)-plane. The algorithm to construct the lower and upper envelopes of a plant u in (q, r)-plane is similar to that given by Rong and Lahdelma (2007a).

After the envelopes of each plant are constructed, we need to construct the envelopes of the system because energy storages affect system-wide energy balance. Based on the characteristic of envelopes, the first and the last points of lower and upper envelopes coincide. To compound the envelopes of two plants, only the superposition of the points on the lower envelopes of two plants can become the candidates of the points on compounded lower envelopes. Similarly, only the superposition of the points on the upper envelopes can become the candidates of the points on the compounded upper envelopes. Therefore, the compounded envelopes of multiple plants can be constructed by sequentially compounding the points on the lower and upper envelopes of the plants. Fig. 2 illustrates the compounded envelope of two plants.

For the trigeneration system, we define $r = \underline{f}_q(q)$ and $r = \bar{f}_q(q)$ as the lower and upper envelopes of (q, r)-region in terms of q and $q = \underline{f}_r(r)$ and $q = \bar{f}_r(r)$ as the lower and upper envelopes of (q, r)-region in terms of r.

4. Solution approaches

4.1. Lagrangian relaxation framework

The basic idea of Lagrangian relaxation (LR) technique is to relax the complicating constraints in the system by using Lagrangian multipliers and formulate a two-level structure. At the lower level, the well-structured Lagrangian subproblems (LS), which are easy to solve, are formed and solved. At the upper level, the Lagrangian multipliers are updated in the direction so that the objective function of Lagrangian dual (LD) problems can be improved. For a successful use of the LR approach, firstly, an appropriate LS formulation has to be constructed. i.e. which constraints are relaxed and how are the constraints relaxed. Secondly, an appropriate non-differentiable optimization techniques (NDO) such as some variants of subgradient based techniques, bundle method (BM) (Redondo and Conejo, 1999; Zhang et al., 1999) and analytical center cutting plane method (ACCPM) (Gondzio et al., 1997) must be employed to solve the LD problem. The employed methods depend on many factors such as problem characteristics and modeling techniques.

4.2. Lagrangian decomposition

In our problem, storage dynamics (6) and partial of the related constraints (8)–(10) are complicating constraints depending on different LS formulation because the storage dynamics introduces the coupling relationship from period to period. For CHP planning with storage, Dotzauer and Ravn (2000) introduced Lagrangian multipliers to directly relax equality storage dynamics (similar to (6)) and adopted the forward projection method to calculate the subgradient and backward projection method to update the Lagrangian multipliers. In their situation, the cost function is represented as the quadratic function of produced energy and the algorithm converges. However, this approach shows slow convergence drawback for our problem. There may be two reasons for this phenomenon. First, the projection method to update the Lagrangian multipliers is in essence a pure subgradient method, which is prone to result in zigzagging phenomenon and crawl toward optimality. Second, the equality constraints are "hard" constraints because the corresponding Lagrangian multipliers associated with the constraints have no restrictions on sign and the oscillation in the LD solution is prone to occur. The effect of above mentioned drawbacks on the quadratic function is weak and serious on the linear cost function.

Here we represent the storage level in each time point t using charge/discharge rate up to period t by eliminating the storage level variables in all of previous periods based on recursive substitution of dynamics (6). Then the relaxation is exercised on constraints (8) and (9) based on the transformed storage level. This approach is similar to that by Guan et al. (1994) but the derivation is a little bit complicated because of the storage loss. Let

$$a_{b,t} = \eta_{b+}x_{b+,t} - \frac{x_{b-,t}}{\eta_{b-}}, \quad b \in B. \tag{12}$$

Based on (6),

$$h_{b,t} = (1-\rho_b)^t h_{b,0} + \sum_{i=0}^{t-1} a_{b,t-i}(1-\rho_b)^i, \quad b \in B,\ t = 1,\dots,T. \tag{13}$$

By substituting the above equation into (9) and (8) we can obtain

$$\bar{h}_b - (1-\rho_b)^t h_{b,0} \leqslant \sum_{i=0}^{t-1} a_{b,t-i}(1-\rho_b)^i \leqslant \underline{h}_b - (1-\rho_b)^t h_{b,0}, \quad b \in B, t = 1,\dots,T-1, \tag{14}$$

and

$$\sum_{i=0}^{T-1} a_{b,T-i}(1-\rho_b)^i = h_{b,T} - (1-\rho_b)^T h_{b,0}, \quad b \in B. \tag{15}$$

We introduce a set of multipliers $\lambda_{b,t}(\geqslant 0), \mu_{b,t}(\geqslant 0), \beta_b$ $(b \in B,\ t = 1,\dots,T-1)$ to relax (14) and (15). Then the cost function (1) becomes

$$\begin{aligned} L = {} & \sum_{t=1}^{T}\left(\sum_{j\in J_t} c_{j,t}x_{j,t} - c_{p,t}x_{p,t}\right) + \sum_{t=1}^{T-1}\sum_{b\in B}\left(\lambda_{b,t}\left(\sum_{i=0}^{t-1} a_{b,t-i}(1-\rho_b)^i + (1-\rho_b)^t h_{b,0} - \bar{h}_b\right)\right. \\ & \left. + \mu_{b,t}\left(\underline{h}_b - \sum_{i=0}^{t-1} a_{b,t-i}(1-\rho_b)^i - (1-\rho_b)^t h_{b,0}\right)\right) + \sum_{b\in B}\beta_b\left(\sum_{i=0}^{T-1} a_{b,T-i}(1-\rho_b)^i + (1-\rho_b)^T h_{b,0} - h_{b,T}\right). \end{aligned} \tag{16}$$

Then we define the stage-wise function $s(\mathbf{x}_t, a_{b,t})$ as

$$s(\mathbf{x}_t, a_{b,t}) = \sum_{j\in J_t} c_{j,t}x_{j,t} - c_{p,t}x_{p,t} + \sum_{b\in B}\left(\sum_{i=t}^{T-1}(1-\rho_b)^{i-t}(\lambda_{b,i}-\mu_{b,i}) + \beta_b(1-\rho_b)^{T-t}\right)a_{b,t}, \quad t = 1,\ldots,T-1, \tag{17}$$

and

$$s(\mathbf{x}_T, a_{b,T}) = \sum_{j\in J_T} c_{j,T}x_{j,T} - c_{p,T}x_{p,T} + \sum_{b\in B}\beta_b a_{b,T}, \tag{18}$$

where vectors $\mathbf{x}_t$ consist of $x_{j,t}$ $(j \in J_t)$ and $x_{p,t}$ in period $t = 1,\ldots,T$.

Define vectors $\boldsymbol{\lambda}_b = [\lambda_{b,1},\ldots,\lambda_{b,T-1}]$, $\boldsymbol{\mu}_b = [\mu_{b,1},\ldots,\mu_{b,T-1}]$, $b \in B$. By using the duality theorem (Bazaraa and Shetty, 1993) and exploiting the decomposable structure in (16) and then regrouping terms in (16) according to periods and using the stage-wise function defined in (17) and (18), a two-level maximum–minimum optimization problem can be formed. The low-level LS can be formed as

$$\begin{aligned}\Phi(\boldsymbol{\lambda}_b, \boldsymbol{\mu}_b, \beta_b) = \text{Min}\quad & L(\boldsymbol{\lambda}_b, \boldsymbol{\mu}_b, \beta_b) \text{ with } L(\boldsymbol{\lambda}_b, \boldsymbol{\mu}_b, \beta_b) = \sum_{t=1}^{T} s(\mathbf{x}_t, a_{b,t})\\ &+ \sum_{b\in B}\left(\sum_{t=1}^{T-1}\left((\lambda_{b,t}-\mu_{b,t})(1-\rho_b)^t h_{b,0} - \lambda_{b,t}\bar{h}_{b,t} + \mu_{b,t}\underline{h}_{b,t}\right) + \beta_b(1-\rho_b)^T h_{b,0} - \beta_b h_{b,T}\right).\end{aligned} \tag{19}$$

Subject to (2)–(5) and (10), (11).

Given $\lambda_{b,t}, \mu_{b,t}, \beta_b, b \in B$, in each stage (period), each stage problem can be solved independently. Min $s(\mathbf{x}_t, a_{b,t})$ can be solved efficiently by the Tri-Commodity Simplex (TCS) algorithm (Rong and Lahdelma, 2005a) specialized for trigeneration planning problem. A generic LP solver can also, in principle, be used to solve the resultant LS. The difference lies in the computational speed. We demonstrate this later in numerical experiments.

Then high-level LD problem can be represented as

$$\begin{aligned}&\text{Max}\quad \Phi(\boldsymbol{\lambda}_b, \boldsymbol{\mu}_b, \beta_b)\\ &\text{Subject to}\quad \boldsymbol{\lambda}_b \geqslant 0,\quad \boldsymbol{\mu}_b \geqslant 0,\quad b \in B.\end{aligned} \tag{20}$$

4.3. Solution to Lagrangian dual problem

Subgradient method is one of the commonly used methods for solving LD problems (Bazaraa and Shetty, 1993). Pure subgradient method suffers from the drawback of zigzagging phenomenon that might results in slow and unsteady convergence. BM (Redondo and Conejo, 1999; Zhang et al., 1999) and ACCPM (Gondzio et al., 1997) can provide search directions better than the subgradient method. However, much more computational effort is needed to obtain the related search directions. In terms of LR method for solving the generic large scale LP problem, average direction strategy (ADS) (Sherali and Ulular, 1989), one of the deflected subgradient optimization methods, can yield an effective and robust scheme for most part. The deflected subgradient method deflects the current subgradient by combining with the previous search direction. The search direction $\mathbf{d}^k$ at current iteration k can be represented as

$$\mathbf{d}^k = \mathbf{g}^k + \psi^k\mathbf{d}^{k-1}, \tag{21}$$

where $\psi^k \geqslant 0$ is a deflection parameter, and $\mathbf{g}^k$ is the current subgradient.

In ADS,

$$\psi^k = \frac{\|\mathbf{g}^k\|}{\|\mathbf{d}^{k-1}\|}. \tag{22}$$

That is, the search direction bisects the angle formed by current subgradient and the previous search direction.

For our problem, we define **d** and **g** as follows.

$$\mathbf{d} = [d_{q,1}, \ldots, d_{q,T-1}, d_{q,T}, d_{q,T+1}, \ldots, d_{q,2T-1}, d_{r,1}, \ldots, d_{r,T-1}, d_{r,T}, d_{r,T+1}, \ldots, d_{r,2T-1}], \tag{23}$$

$$\mathbf{g} = [g_{q,1}, \ldots, g_{q,T-1}, g_{q,T}, g_{q,T+1}, \ldots, g_{q,2T-1}, g_{r,1}, \ldots, g_{r,T-1}, g_{r,T}, g_{r,T+1}, \ldots, g_{r,2T-1}]. \tag{24}$$

Where the components of $\boldsymbol{g}$ are computed based on (20) associated with (16)–(19)

$$g_{b,t} = \sum_{i=0}^{t-1} a_{b,t-i}(1-\rho_b)^i + (1-\rho_b)^t h_{b,0} - \bar{h}_b, \quad b \in B, \quad t = 1, \ldots, T-1, \tag{25}$$

$$g_{b,T} = \sum_{i=0}^{T-1} a_{b,t-i}(1-\rho_b)^i + (1-\rho_b)^T h_{b,0} - h_{b,T}, \quad b \in B, \tag{26}$$

$$g_{b,t+T} = \underline{h}_b - \sum_{i=0}^{t-1} a_{b,t-i}(1-\rho_b)^i - (1-\rho_b)^t h_{b,0}, \quad b \in B, \quad t = 1, \ldots, T-1. \tag{27}$$

Then the Lagrangian multipliers for the next iteration $k+1$ $\lambda_{b,t}^{k+1}$, $\mu_{b,t}^{k+1}$, β_b^{k+1} can be updated by moving along the search direction with step-size ξ^k ($\xi^k > 0$).

$$\lambda_{b,t}^{k+1} = \max\{0, \lambda_{b,t}^k + \xi^k d_{b,t}^k\}, \quad b \in B, \quad t = 1, \ldots, T-1, \tag{28}$$

$$\beta_b^{k+1} = \beta_b^k + \xi^k d_{b,T}^k, \quad b \in B, \tag{29}$$

$$\mu_{b,t}^{k+1} = \max\{0, \mu_{b,t}^k + \xi^k d_{b,T+t}^k\}, \quad b \in B, \quad t = 1, \ldots, T-1. \tag{30}$$

Step-length rules also play an important role in governing both the ultimate convergence and the rate of convergence to the optimality. We adopted the following rules for updating step-size ξ. If there is no improvement for the LD solution for a given number of iterations, the current ξ is decreased by multiplying a degrading factor γ $(0 < \gamma < 1)$. Now we summarize the procedures for solving the LD problem.

Algorithm 1. Deflected subgradient based LR algorithm

Step 0. Initialization
Degrading factor γ, small positive values ε and δ; Best dual value $z^{bd} = -M$ (M is a large positive number); Best solution vector $\mathbf{x}^* = \emptyset$ for decision variables; Given improvement iterations l_0 ; improvement iteration counter $i = 0$; LD iteration counter $k = 1$; Initial multipliers $\lambda_{b,t}^1, \mu_{b,t}^1, \beta_b^1$ ($b \in B$, $t = 1, \ldots T-1$), step-size ξ^1, and search direction vector $\mathbf{d}^0 = \mathbf{0}$.

Step 1. Calculate dual function $\Phi(\boldsymbol{\lambda}_b^k, \boldsymbol{\mu}_b^k, \beta_b^k)$ by solving (19).

Step 2. Termination condition
if $(|\Phi(\boldsymbol{\lambda}_b^k, \boldsymbol{\mu}_b^k, \beta_b^k) - z^{\mathrm{bd}}| < \delta$ or $\xi^k < \varepsilon)$
  go to end;
endif

Step 3. Update best dual solution and step-size ξ^k
if$(\Phi(\boldsymbol{\lambda}_b^k, \boldsymbol{\mu}_b^k, \beta_b^k) > z^{\mathrm{bd}})$
  $z^{\mathrm{bd}} \leftarrow \Phi(\boldsymbol{\lambda}_b^k, \boldsymbol{\mu}_b^k, \beta_b^k)$
  update $\mathbf{x}^*$
  $i = 0$
else
  $i \leftarrow i + 1$
endif
if $(i \geqslant l_0)$
  $\xi^k \leftarrow \gamma \xi^k$
endif

Step 4. Calculate subgradient $\mathbf{g}^k$ based on (25)–(27)
Step 5. Calculate search direction $\mathbf{d}^k$ based on (21) and (22)
Step 6. Update multipliers based on (28)–(30).
Step 7. $k \leftarrow k+1$, go to **Step 1**

4.4. Obtaining feasible solutions

In general, the LD solution is associated with an infeasible solution because some of the relaxed constraints cannot be satisfied. In the LS formulation, we relax the boundary constraints and terminal conditions for storages. Therefore, in the infeasible solution, the storage level can be lower than lower bound (generally zero) or higher than upper bound (capacity) of the storages or the terminal conditions for storages cannot meet. A heuristic method is developed to obtain a good feasible solution by considering the dual price of energy production, energy loss and the charge/discharge efficiency of the storage.

In the LD solution, let $x^{(1)}_{b+,t}/x^{(1)}_{b-,t}$ ($b \in B$) represent the charge/discharge rate in each period t , $\alpha_{q,t}$ and $\alpha_{r,t}$ the dual price of energy balances (4) and (5). The physical meaning of dual price for (4) and (5) are the marginal cost for producing q- and r-commodity respectively. For restoring the feasibility of the storage level cost-efficiently, we need to determine the priority order of periods to absorb energy surplus or supply energy lack. The relative value of energy production cost is more meaningful than the absolute value when there is energy storage loss. We use *weighted dual price* (WDP) $\alpha'_{q,t}$ and $\alpha'_{r,t}$ as the measure of relative production cost of q- and r-commodity in period t. WDP is determined by considering the energy loss and charge/discharge efficiency

$$\begin{cases} \alpha'_{b,t} = (1-\rho_b)^t \alpha_{b,t}\eta_{b-,t} & \text{if } x^{(1)}_{b-,t} > 0, \\ \alpha'_{b,t} = (1-\rho_b)^t \alpha_{b,t}/\eta_{b+,t} & \text{if } x^{(1)}_{b+,t} > 0 \end{cases}. \tag{31}$$

When both $x^{(1)}_{b-,t}$ and $x^{(1)}_{b+,t}$ equal zero, then the WDP is calculated based on the first formula in (31) if period t is chosen as the adjustment period for increasing discharge and the second formula for increasing charge.

The above relationship means that the energy loss can be explained as either the loss of storage volume or loss of monetary value. Intuitively, the stored energy should be released as early possible when WDP is higher. The lack energy should be supplied by the production and storage in later periods when WDP is lower. When the storage level needs increasing, the periods with lowest WDP should be given priority while the periods with the highest WDP should be given priority when the storage level needs decreasing. The following heuristic procedures are designed based on these intuitive ideas. If both q- and r-storages are active, we use sequential strategy to restore the feasibility. First the feasibility for q-storage is restored, then r-storage, or vice versa.

Algorithm 2. Heuristic procedures for obtaining a good feasible solution.

Step 1. Primal feasibility is restored based on the feasible conditions concerning the storage dynamics (6)–(10) and feasible regions represented explicitly by the compounded system envelope (Section 3).

for (t := 1 to T**)**
  if (storage level in period t **is infeasible)**
    (1) Sort the periods before t based on the increasing (decreasing) order of WDP if the storage level is less than zero (greater than the capacity in period t).
    (2) Check the sorted periods in sequence, for each selected period, as much charge/discharge amount as possible is exercised under the following four conditions: (i) the adjustment cannot introduce new infeasible periods before t ; (ii) the adjustment cannot violate the charge/discharge limits (constraints (10)) ; (iii) the adjustment cannot go beyond the feasible region of the trigeneration system (Section 3) and (iv) no matter how many periods need adjustment, the combined adjustment is just enough for restoring the feasibility in period t.
  end if
end for

Step 2. The feasible solution in **Step 1** is further improved by exercising the forced discharge to remove the redundant energy storage.

for (**t** := **1 to** T)

if (t is a period satisfying the following conditions : (i) non-zero charge/discharge period t; and (ii) after period t, there is no charge/discharge activity for a given number of periods but the storage levels are non-zero in these periods)

Forced discharge is exercised immediately after t to remove the redundant storage.

end if

end for

5. Numerical experiments

To test the performance of the deflected subgradient based Lagrangian relaxation (LR) algorithm for trigeneration planning with storages, we implement the LR algorithm using C++ and Microsoft Visual Studio with the efficient Tri-Commodity Simplex (TCS) (Rong and Lahdelma, 2005a) as a solver for the low-level Lagrangian subproblems (LS). The planning problems are solved on hourly basis for different planning horizons: weekly (168-hour), monthly (672-hour) and yearly (8760-hour). For a sub-case of weekly planning problem, we also solve the planning model (1)–(11) directly using an efficient sparse Simplex code LP2 (Lahdelma et al., 1986; Ruuth et al., 1985) to obtain the true optimum so that both the computational time and optimality of the LR algorithm can be compared with that of the standard LP solver. For time comparison, we mention the relative speed of several LP solvers. For solving the small-size hourly model, TCS is 64 times faster than LP2 and LP2 is 46 times (Lahdelma and Hakonen, 2003; Rong, 2006; Rong and Lahdelma, 2007a) faster than ILOG CPLEX 9.0 (the detailed settings of the CPLEX solver are referred to Rong and Lahdelma, 2007a), one of the widely used commercial solvers for the large-size problem. All test runs are performed in a 2.2 GHz Pentium 4 PC (512 Mb RAM) under the Windows XP operating system.

5.1. Test problems

Our test problems are based on real-life power plant models. The trigeneration plant models adopted here are similar to those in Rong and Lahdelma (2005a). In our test models, the number of characteristic points varies from 10 to 70 per production plant. The production facilities include trigeneration plants and separate energy production units (including contracts). A total of six production models are generated. Table 2 shows the structure of the production models and dimensions of planning problems for different planning horizons. TRIs and Non-TRIs represent the number of trigeneration plants and separate energy production units, respectively. m_h (number of constraints) $\times n_h$ (number of variables) is the size of the *hourly* model without dynamic constraints (refer to (2)–(5)), where $m_h = \text{TRIs} + \text{Non-TRIS}{+}3$, $n_h = |J| + 5$ ($|J|$ is the total number of extreme points for all of the plants). The size of the *hourly* Lagrangian subproblem after the dynamic constraints are relaxed is $m_h \times (n_h{+}4)$ (refer to (19)). m (number of constraints) $\times\, n$ (number of variables) is the size of the overall planning problem with both q- and r-storages active. Based on models (1)–(11), $m = (m_h + 2)T$ and $n = (n_h + 6)T$, the number of multipliers used to form the Lagrangian dual problem (16) is

Table 2
Structure of production models and dimensions of the planning problems for different planning horizons

Model	TRIs	Non-TRIs	m_h	n_h	m			n		
					Weekly	Monthly	Yearly	Weekly	Monthly	Yearly
A1	1	0	4	72	1008	4032	52,560	13,104	52,416	683,280
A2	2	1	6	109	1344	5376	70,080	19,320	77,280	1,007,400
A3	3	2	8	183	1680	6720	87,600	31,752	127,008	1,655,640
A4	5	3	11	220	2184	8736	113,880	37,968	151,872	1,979,760
A5	8	3	14	270	2688	10,752	140,160	46,368	185,472	2,417,760
A6	10	3	16	364	3024	12,096	157,680	62,160	248,640	3,241,200

Table 3
Parameters of q- and r-storages

Parameter	q-Storage	r-Storage
$\bar{h}$	100	90
$\bar{x}$	20	15
η	0.98	0.95
ρ	0.04	0.06

$2 \times 2 \times (T-1) + 2 = 4T - 2$, where T is the number of hours over the planning horizon. Table 3 shows the parameters of q- and r-storages, where $\bar{h}$, $\bar{x}$, η, and ρ denote the capacity (MW), maximum charge/discharge rate (MW/hour), charge/discharge efficiency and energy loss of the storage, respectively. The storage parameters are same for all of the test problems.

To form valid test problems, we generate q- and r-commodity demand data based on the history data of a Finnish energy company and power price based on the history data of Nord Pool (Nordic power exchange) (Nord Pool, 2004).

5.2. Computational results

For measuring the optimality performance of the LR algorithm, three gaps are introduced: GAP, GAP0 and GAP1. $\mathrm{GAP} = |100(z^{\mathrm{bd}} - z^{\mathrm{fea}})/z^{\mathrm{bd}}|$ (percentage), $\mathrm{GAP0} = |100(z^{\mathrm{bd}} - z^{\mathrm{opt}})/z^{\mathrm{opt}}|$ (percentage) and $\mathrm{GAP1} = |100(z^{\mathrm{opt}} - z^{\mathrm{fea}})/z^{opt}|$ (percentage), where z^{opt}, z^{bd}, and z^{fea} represent the true optimal solution, LD solution and feasible solution respectively. For minimization problem, $z^{\mathrm{bd}} \leqslant z^{\mathrm{opt}} \leqslant z^{\mathrm{fea}}$. GAP is an approximate gap measure of the feasible solution based on the LD solution. GAP0 measures the tightness of the LD solution as a lower bound on the optimal solution. GAP1 is a strict gap measure of the feasible solution based on the true optimal solution. All of these gap measures can reflect the convergence property of the LR algorithm. However, in practice, GAP is generally used because it is difficult to obtain the true optimal solution for large-size (monthly, yearly, and multi-year) problems.

We solve the planning problems for different horizons by the LR algorithm. For weekly (168-hour) planning, we take the average of the results for 8 sub-cases (weeks 1–4 and weeks 20–23). For monthly planning, we take the average of the results for 4 sub-cases (weeks 1–4, weeks 5–8, weeks 20–23, and weeks 24–27). January 1 is the first day of the week one. To reduce the effect of variation in CPU time, each sub-case of test problems was run ten times and average solution time (CPU time) is computed. Tables 4 and 5 show the performance for solving weekly, monthly for either q- or r-storage active or both q- and r-storages active, respectively. Table 6 shows the yearly planning for either q- or r-storage active or both q- and r-storages active or no storage active.

From Tables 4–6, the GAP measure for solving the problem is very small. The average gap for solving weekly, monthly and yearly planning problems are 0.048%, 0.028% and 0.024%. This means that the LR algorithm converges and the near optimal-solution can be obtained. LDiters (number of iterations to terminate the solution process of the LD problem) reflects the convergence speed of the LR algorithm. LDiters increases as

Table 4
Performance of the LR algorithm for weekly planning problems

Model	Only q-storage active			Only r-storage active			q- and r-storages active		
	LDiters	CPU (seconds)	GAP (%)	LDiters	CPU (seconds)	GAP (%)	LDiters	CPU (seconds)	GAP (%)
A1	60.1	0.035	0.085	40.6	0.026	0.032	62.4	0.045	0.210
A2	57.0	0.043	0.098	40.3	0.028	0.025	68.5	0.055	0.063
A3	64.3	0.102	0.077	42.3	0.069	0.038	67.3	0.123	0.059
A4	54.3	0.130	0.117	48.1	0.123	0.016	64.5	0.169	0.029
A5	57.8	0.160	0.032	43.5	0.113	0.002	63.6	0.179	0.016
A6	57.6	0.332	0.037	44.8	0.264	0.007	62.0	0.369	0.033
Avg.	58.7	0.094	0.078	43.0	0.072	0.018	65.3	0.114	0.049

Table 5
Performance of the LR algorithm for monthly planning problems

Model	Only q-storage active			Only r-storage active			q- and r-storages active		
	LDiters	CPU (seconds)	GAP (%)	LDiters	CPU (seconds)	GAP (%)	LDiters	CPU (seconds)	GAP (%)
A1	66.5	0.159	0.125	55.0	0.136	0.031	62.5	0.188	0.243
A2	81.0	0.229	0.021	48.0	0.132	0.063	79.8	0.247	0.032
A3	85.3	0.550	0.039	61.8	0.424	0.018	72.8	0.548	0.076
A4	75.5	0.723	0.028	56.5	0.533	0.002	72.3	0.701	0.022
A5	79.8	0.850	0.006	40.5	0.432	0.011	76.3	0.841	0.011
A6	76.5	1.748	0.012	52.5	1.162	0.003	73.5	1.737	0.035
Avg.	77.4	0.710	0.024	52.4	0.470	0.017	72.8	0.710	0.042

Table 6
Performance of the LR algorithm for yearly planning problems

Model	Only q-storage active			Only r-storage active			q- and r-storages active			No storage
	LDiters	CPU (seconds)	GAP (%)	LDiters	CPU (seconds)	GAP (%)	LDiters	CPU (seconds)	GAP (%)	CPU (seconds)
A1	119	3.680	0.041	71	2.355	0.030	148	5.642	0.053	0.026
A2	132	4.883	0.009	118	4.438	0.005	150	6.385	0.011	0.032
A3	150	12.435	0.014	87	7.549	0.013	120	11.471	0.022	0.077
A4	115	11.969	0.011	90	9.214	0.001	132	14.641	0.017	0.122
A5	100	12.455	0.004	76	9.219	0.000	77	10.096	0.007	0.121
A6	60	16.777	0.177	96	26.633	0.001	101	29.225	0.007	0.287
Avg.	112.7	10.366	0.052	89.7	9.901	0.005	121.3	12.910	0.014	0.111

the planning horizon increases based on Tables 4–6. But the increase factors are within a very reasonable range, less than two from weekly to yearly planning on the average. That means, the convergence speed of the algorithm is not much sensitive to the planning horizon. The average solution times for the weekly, monthly and yearly planning problem are 0.093, 0.63 and 11 seconds. The maximum solution time to the largest generated yearly planning problem is about 30 seconds. This is sufficiently fast to make several advanced analyses feasible, e.g. risk analysis (Makkonen and Lahdelma, 1998, 2001; Rong and Lahdelma, 2005b, 2007b) based on scenario analysis (stochastic simulation). Based on Table 6, LDiters approximates the ratio of the CPU time for solving the problem with storage and without storage. This means that the computational requirement for finding the search direction and updating the Lagrangian multipliers is very small.

Next, we compare the performance of the LR algorithm against a generic LP solver for solving models (1)–(11) directly. Table 7 shows the performance of the LR algorithm against LP2 for a sub-case (week 1) of weekly planning problems with active q- and r-storages. We can see that the LD solution provides a very tight bound on the optimal solution based on GAP0 measure. This again indicates that the convergence property of

Table 7
Performance of the LR algorithm against LP2 for a sub-case of weekly planning problems with active q- and r-storages

Model	LP2		LR			GAP0 (%)	GAP1 (%)
	CPU (seconds)	z^{opt}	CPU (seconds)	z^{bd}	z^{fea}		
A1	17.8	−212909	0.053	−212923	−211425	0.00675	0.69683
A2	69.0	−654202	0.055	−654208	−654195	0.00086	0.00110
A3	85.7	−567720	0.119	−567732	−567718	0.00213	0.00040
A4	234.3	−829521	0.177	−829567	−829343	0.00553	0.02149
A5	347.5	−969699	0.199	−969708	−969624	0.00089	0.00772
A6	627.8	−1041895	0.449	−1041937	−1041759	0.00401	0.01309
Avg.	230.3	−712658	0.175	−712679	−712344	0.00300	0.04403

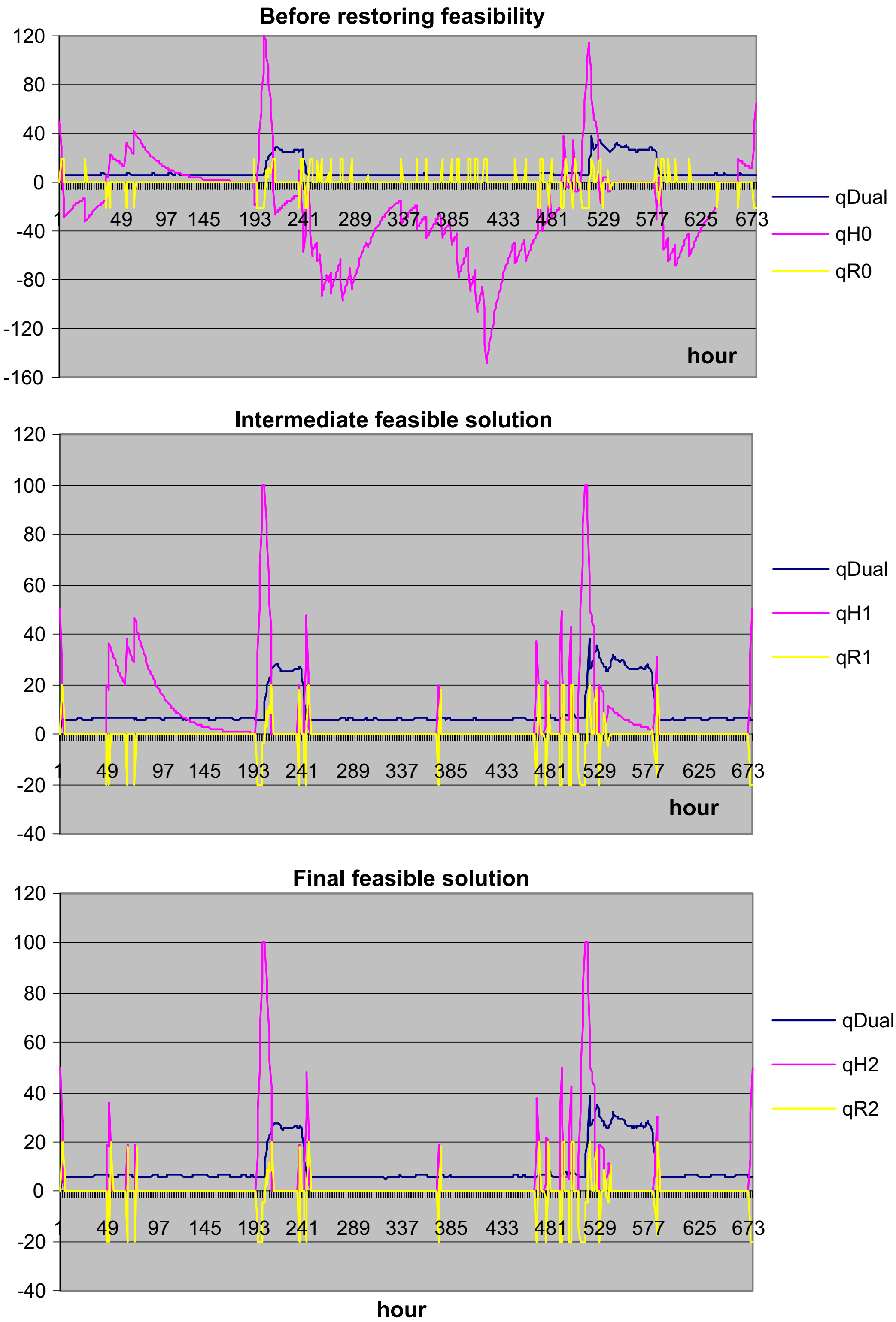

Fig. 3. Storage dynamics (the adjustment of q-storage level (qH) (MW) and the charge/discharge rate (qR) (MW/hour) during the process of restoring feasibility from the LD solution). qDual (Euros/MW): dual price of q-energy balance from the LD solution. The number attaching to qH and qR represents the change of qH and qR. Negative qR represents charging the storage.

the LR algorithm is good. Based on small GAP1 measure, we can view the LR solution as the optimal solution in most cases. GAP1 measure of A1 is a little large, it can be viewed as a good near-optimal solution. A1 contains only one trigeneration plant (Table 2) and the adjustment flexibility is relatively poor. For the computational speed, the LR algorithm is 1316 faster than LP2 solver for a weekly planning problem (a few thousand constraints and a few ten thousand variables (Table 2)). As the planning horizon increases, the speed ratio of the LR algorithm against LP2 can be even larger. Generally it is difficult for a generic LP solver to solve the large-size problems within the reasonable time limits. Therefore, it is imperative that the decomposition technique should apply for the long-term planning problem.

Finally, we illustrate storage dynamics and the heuristic procedures for restoring feasibility using a sub-case of monthly planning problems. Fig. 3 shows the adjustment of q-storage level (qH) (MW) and charge/discharge rate (qR) (MW/hour) (negative value represents charging the storage) during the process of restoring the feasibility after the LD problem was solved. In the meanwhile, we show the profile of the dual price (qDual) (Euros/MW) of the q-energy balance for the LD solution so that we can analyze the relationship between the dual price and charging/discharging process. qH0 and qR0 represent the q-storage level and charge/discharge rate for the LD solution. qH1 and qR1 represent the results after the first step of the heuristic procedures (Section 4.4). qH2 and qR2 represent the results after the second step of the heuristic procedures (Section 4.4). We can see that the LD solution (qH0, qR0) provides a roughly reasonable but an infeasible schedule. There are two periods (hours 199 — 237 and hours 511—577) with higher dual prices (marginal production cost for the q-commodity). Before the periods with higher dual price begin, the storage are charged to full (qH0 reaches the highest level) with the energy produced at lower cost and discharged (qH0 is reduced) after the periods with higher price begin. That is, the incorporation of the storage can partially alleviate the higher cost for providing the energy commodity and increase the flexibility of the system operation. However, we can see that the schedule is infeasible (e.g. qH0) exceeds the storage capacity (100 MW) around hour 200 and 515 because of the overcharge and falls below zero between 3–44 hours, 207–494 hours and 575–654 hours because of the overdischarge. The primal feasibility (qH1 and qR1) is restored after the first step of the heuristic procedures. But it seems that there are some periods (hours 45–187) with unnecessary non-zero storage level (qH1). Then the forced discharge (the second step of the heuristic procedures) is exercised to empty the storage and a better feasible solution (qH2, qR2) can be obtained.. For the illustrated monthly planning problem, the objective functions of the LD solution, intermediate and final feasible solutions are -1994023, -1991004 and -1991620, respectively.

6. Conclusions

We have solved the Lagrangian relaxation of the long-term trigeneration planning problem with the energy storage by using a deflected subgradient method. Several considerations contribute to the efficiency and effectiveness of the algorithm: the proper strategy to relax the dynamic constraints (by transformation), the good convergence property of deflected subgradient method, the heuristic for restoring the feasibility, and the efficient tri-commodity simplex (TCS) for solving the Lagrangian subproblems. In the test run with realistic trigeneration production models for weekly (168-hour), monthly (672-hour) and yearly (8760-hour) planning model, the presented LR algorithm shows robust and fast convergence speed based on the number of iterations for solving the Lagrangian dual problem and high solution quality based on both the dual gap and gap between the feasible solution and the optimal solution. This lays solid foundation for the scenario-based risk analysis (Makkonen and Lahdelma, 1998, 2001; Rong and Lahdelma, 2005b, 2007b) because both solution time and solution quality are critical when multiple randomly generated scenarios are solved.

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